CHAPTER 5

PRODUCTION AND COST

CHAPTER SUMMARY

This chapter presents a basic economic analysis of production and cost. Primary topics include production functions, optimal input choice, cost, and profit maximization. The chapter also provides an introduction to cost estimation and factor demand curves. The chapter includes a short case study (Rich Manufacturing). An appendix derives the factor-balance equation.

CHAPTER OUTLINE

PRODUCTION FUNCTIONS
  - Returns to Scale
    - Managerial Application: Increasing Returns to Scale at Volkswagen
  - Returns to a Factor
    - Managerial Application: Studying for an Exam—the Law of Diminishing Returns
    - Managerial Application: Baseballs Batting Averages

CHOICE OF INPUTS
  - Production Isoquants
    - Managerial Application: Substitution of Inputs in Home Building
  - Isocost Line
    - Managerial Application: General Motors is Shanghaied
  - Cost Minimization
  - Changes in Input Prices
    - Academic Application: Minimum Wage Laws

COSTS
  - Cost Curves
    - Managerial Application: Job Seekers Use Internet
    - Production Functions and Cost Curves
      - Managerial Application: Industry Responds to Higher Metals Prices
  - Opportunity Costs
  - Short Run Versus Long Run
    - Fixed and Variable Costs
    - Short-Run Cost Curves
    - Long-Run Cost Curves
    - Managerial Application: DeLorean Automobiles
    - Managerial Application: Public Utilities
Minimum Efficient Scale
Managerial Application: Size Doesn’t Always Matter

Learning Curves
Academic Application: Economies of Scale and Learning Effects in the Chemical Processing Industry

Economies of Scope
Managerial Application: Economies of Scale and Scope in Apartment Management
Managerial Application: Economies of Scale and Scope in DSP Production

PROFIT MAXIMIZATION

FACTOR DEMAND CURVES
Managerial Application: China Becomes the “World’s Smokestack”
Managerial Application: Hog Producers React to Increase in Corn Prices
Managerial Application: Demand for Labor Falls Following 9/11 Terrorist Attacks

COST ESTIMATION

SUMMARY

APPENDIX: THE FACTOR-BALANCE EQUATION
Slope of an Isoquant
Factor-Balance Equation

TEACHING THE CHAPTER

Chapter 5 reviews the basics of production and costs. Although most students will likely have been exposed to the concepts of short-run and long-run costs of production in previous courses, the graphical and quantitative analysis using isoquants and isocost lines may be new for some students. Students typically grasp these concepts more quickly than the consumer choice tools presented in chapter 2, however, instructors will want to review the key concepts and several quantitative examples to be sure students understand the material. In particular, returns to scale are given extensive coverage in the chapter and students are less likely to be familiar with these concepts. Instructors will need to spend varying amounts of time on this chapter based on the backgrounds of the students in the course and the goals of the course.

There are numerous Self-Evaluation Problems and Review Questions that can be assigned to determine whether students understand the concepts and quantitative tools presented in the chapter. Instructors will likely want to review several of these questions before assigning the Analyzing Managerial Decisions scenarios. There are also numerous Managerial Applications that have non-technical examples of the concepts that can be used to generate class discussion.
There are three *Analyzing Managerial Decisions* scenarios presented in this chapter. The first, “Choosing the Mix of People and Machines to Ticket Airline Customers”, is a quantitative scenario asking students to calculate and graph the costs of production. The problem then also asks students consider what other factors should be considered before making a decision about the optimal mix of inputs. In the second scenario, “Developing Economies of Scale for Malaysia’s Proton Holdings”, students must apply the concept of economies of scale and determine what actions the company needs to take to achieve economies of scale. The third scenario, “Rich Manufacturing”, asks students to evaluate how a manager should respond to a price increase that results from a pricing policy of cost-plus pricing. Students are asked to consider both short run and long run implications. (See the Solutions Manual for the answers to these problems).

**REVIEW QUESTIONS**

5–1. Distinguish between returns to scale and returns to a factor.

*Returns to scale refers to the relation between output and a proportional variation of all inputs taken together. Returns to a factor refers to the relation between output and the variation in only one input, holding all other inputs fixed.*

5–2. Your company currently uses steel and aluminum in a production process. Steel costs $.50 per pound, and aluminum costs $1.00 per pound. Suppose the government imposes a tax of $.25 per pound on all metals. What affect will this have on your optimal input mix? Show using isoquants and isocost lines.

*It is likely to shift the optimal mix toward using more aluminum. The $0.25 per pound tax is a larger percentage tax on steel. Thus, the price of steel has increased relative to the price of aluminum. The graphic analysis is similar to Figure 5.7. The original isocost line has a slope of $-\frac{.5}{1} = -0.5$. The slope of the isocost line after the tax is $-\frac{(0.5+0.25)}{(0.5+0.25)} = -0.6$. The optimal input mix to produce any given quantity of output now uses more aluminum and less steel. Note: The chapter concentrates on the substitution effect and does not go into detail about scale effects. In a more in-depth analysis, one would consider changes in the scale of output that also might accompany an increase in input prices.*

5–3. Your company currently uses steel and aluminum in a production process. Steel costs $.50 per pound, and aluminum costs $1.00 per pound. Suppose that inflation doubles the price of both inputs. What affect will this have on your optimal input mix? Show using isoquants and isocost lines.
If inflation has the same percentage effect on both inputs, there is no change in relative prices. Thus, there is no change in the optimal input mix to produce any given output. A graphical picture such as Figure 5.7 remains unchanged.

5–4. Is the "long-run" the same calendar time for all firms? Explain.

No. The short run is the operating period during which at least one input is fixed in supply. In the long run, no inputs are fixed. These definitions are not based on calendar time. Rather, the length of each period depends on how long it takes the firm to vary all inputs. This time can vary across firms.

5–5. You want to estimate the cost of materials used to produce a particular product. According to accounting reports, you initially paid $50 for the materials that are necessary to produce each unit. Is $50 a good estimate of your current production costs? Explain.

The historical cost is not necessarily a good estimate of current production cost. The relevant cost is the current opportunity cost of the materials.

5–6. Suppose that average cost is minimized at 50 units and equals $1. What is marginal cost at this output level?

It is $1. Marginal equals average when the average is at a minimum.

5–7. What is the difference between economies of scale and economies of scope?

Economies of scale involve efficiencies from producing higher volumes of a given product, while economies of scope involve cost savings that result from joint production.

5–8. What is the difference between economies of scale and learning effects?

Economies of scale imply reductions in average cost as the quantity being produced in the production period increases. Learning effects imply a shift in the entire average cost curve (the average cost for producing a given quantity in a production period decreases with cumulative volume).

5–9. Suppose that you can sell as much of a product as you want at $100 per unit. Your marginal cost is: MC = 2Q. Your fixed cost is $50. What is the optimal output level? What is the optimal output, if your fixed cost is $60?
Setting the marginal revenue of $100 equal to the marginal cost, yields an optimal output of 50 units. The optimal output does not depend on the fixed cost. In either case, it makes sense to continue to operate and the optimal output is 50.

5–10. Discuss two problems that arise in estimating cost curves.

Statistical problems, such as omitted-variables problems (discussed in chapter 4) can arise in cost estimation. Among the most common problems in cost estimation are difficulties in obtaining data on the relevant costs. Accounting costs are often poor estimates of the opportunity costs of resources.

5–11. Suppose that the marginal product of labor is: \( MP = 100 - L \), where \( L \) is the number of workers hired. You can sell the product in the marketplace for $50 per unit and the wage rate for labor is $100. How many workers should you hire?

You want to hire workers up to the point where the marginal revenue product equals the wage rate. The marginal revenue product in this example is \( 50 \times MP = 5000 - 50L \). Setting this expression equal to $100 and solving for \( L \) indicates that it is optimal to hire 98 workers. The 98th worker brings $100 of incremental revenue into the firm. This figure equals the marginal cost of the employee ($100).

5–12. Textbook writers typically receive a simple percentage of total revenue generated from book sales. The publisher bears all the production costs and chooses the output level. Suppose the retail price of a book is fixed at $50. The author receives $10 per copy, and the firm receives $40 per copy. The firm is interested in maximizing its own profits. Will the author be happy with the book company’s output choice? Does the selected output maximize the joint profits (for both the author and company) from the book?
The author will not be happy with the company’s output choice. The author wants to maximize the number of copies sold (he does not care about the costs of production). Alternatively, the publisher will want to produce an output where its marginal revenue of $40 equals its marginal cost of production. Thus, under the current contract, the author would prefer a higher output than the publisher. The selected output does not maximize joint profits. To maximize joint profits, the output should be selected where the marginal revenue for both the author and the publisher of $50 is equal to the marginal cost. From the standpoint of joint profit maximization, the publisher selects too low a level of output. This is because the publisher bears all the incremental cost, but only receives a fraction of the incremental revenue.

5–13. Suppose your company produces one product and that you are currently at an output level where your price elasticity is 0.5. Are you at the optimal output level for profit maximization? How can you tell?

No you are not producing an optimal output. Profit maximization cannot occur at a point where the price elasticity is less than 1 (demand is inelastic). Total revenue will increase if output is reduced. Total costs will also decrease. Therefore, profits will rise with a reduction in output (over some range).

5–14. Semiconductor chips are used to store information in electronic products, such as personal computers. One of the early leaders in the production of these chips was Texas Instruments (TI). During the early period in the development of this industry, TI made the decision to price its semiconductors substantially below its production costs. This decision increased sales, but resulted in near-term reductions in profits. Explain why TI might have made this decision.

TI might have wanted to increase sales and thus its cumulative production to obtain a reduction in production costs through learning effects. These learning effects might give TI a competitive advantage over competitors and thus increase future profits.
5–15. The AFL-CIO has been a steadfast proponent of increasing the minimum wage. Offer at least two reasons why they might lobby for such increases.

One reason is that they might simply be in favor of increasing wages for some minimum wage workers. This support might increase union support among labor and result in future union growth. A second reason is that they favor the increase because it increases the price of low cost non-unionized labor. Increasing the price of this input substitute will increase the demand for higher priced/skilled unionized labor.

5–16. Mountain Springs Water Company produces bottled water. Internal consultants estimate the company’s production function to be \( Q = 300L^2K \), where \( Q \) is the number of bottles of water produced each week, \( L \) is the hours of labor per week, and \( K \) is the number of machine hours per week. Each machine can operate 100 hours a week. Labor costs $20/hour, and each machine costs $1000 per week.

a. Suppose the firm has 20 machines and is producing its current output using an optimal \( K/L \) ratio. How many people does Mountain Springs employ? Assume each person works 40 hours a week.

At optimum:
\[
\frac{\text{MP}_L}{w} = \frac{\text{MP}_K}{r}
\]

\[
600LK = 20
\]
\[
300L^2 = 10
\]

\[
\frac{2K}{L} = 2
\]

If the firm has 20 machines, \( K = 2000 \) hours/week

\[
2000 = L
\]

\[
L = 2000 \text{ hours/week}
\]

\[
L = 50 \text{ people}
\]

b. Recent technological advancements have caused machine prices to drop. Mountain Springs can now lease each machine for $800 a week. How will this affect the optimal \( K/L \) ratio (i.e., will the optimal \( K/L \) ratio be smaller or larger)? Show why.

\[
\frac{2K}{L} = 20
\]

\[
\frac{L}{8}
\]

\[
K = 5
\]

\[
\frac{L}{4}
\]
Previously Mountain Springs had a K/L ratio of 1. Now, it has increased to 5/4. Thus, the decrease in the price of capital caused the optimal K/L ratio to increase.

5–17. The Workerbee Company employs 100 high school graduates and 50 college graduates at respective wages of $10 and $20. The total product for high school graduates is $1000 + 100Q_H$, whereas the total product for college graduates is $5000 + 50Q_C$. $Q_H$ = the number of high school graduates, while $Q_C$ = the number of college graduates. Is the company hiring the optimal amount of each type of worker? If not, has it hired too many high school or too many college graduates? Explain.

No, the MP/Input price ratios for high school and college graduates is as follows: $100/10 = 10$ and $50/20 = 2.5$. Thus high school graduates yield 10 units per dollar, while the college graduates yield 2.5 units per dollar (on the margin). The company should fire college graduates and hire high school graduates.

5-18.

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5-8
a. Complete the above table.

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b. Graph TC, TFC, TVC, MC, AC, AFC, and AVC against Q.

The graphs for TC, TFC, and TVC can be included on the same figure. AC, AFC, and AVC should be on a separate graph since they are dollars/unit (versus total dollars).

5-19. Suppose the Jones Manufacturing Company produces a single product. At its current input mix the marginal product of labor is 10 and the marginal product of capital is 20. The per unit price of labor and capital are $5 and $10, respectively. Is the Jones Company using an optimal mix of labor and capital to produce its current output? If not, should it use more capital or labor? Explain.

The company is using the optimal mix of labor and capital to produce its current output. At an optimal input mix the marginal-product-to-price ratios are equal for all inputs. This condition implies that you cannot gain by spending less on one input and more on another (the marginal output per dollar is the same for all inputs). In this problem the ratios for both inputs is 2 units/dollar.
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5-20. Suppose the production function of PowerGuns Co. is given by

\[ Q = 25LK \]

where \( Q \) is the quantity of guns produced in the month, \( L \) is the number of workers employed, and \( K \) is the number of machines used in the production. The monthly wage rate is $3,000 per worker and the monthly rental rate for a machine is $6,000. Currently PowerGuns Co. employs 25 workers and 40 machines. Assume perfect divisibility of labor and machines.

a. What is the current average product of labor for PowerGuns Co.? What is the current marginal product of machines? (Assume 1 unit increase in machines.)

The current input mix is \( L=25 \) and \( K=40 \), so the current output level is

\[ Q = 25 \times 25 \times 40 = 25000 \]

\[ AP(L) = \frac{Q}{L} = \frac{25000}{25} = 1000 \]

To compute the marginal product of machine, we keep the employment of labor constant, \( L=25 \), and increase \( K \) by 1 unit, so \( K=41 \), and compute the new output level, \( Q = 25 \times 25 \times 41 = 25625 \), thereby the marginal product of machine, \( MP(K) = \frac{25625 - 25000}{1} = 625 \).

b. Does PowerGuns' production function display increasing, decreasing, or constant returns to scale? Explain.

The production function displays increasing returns to scale. For example, if PowerGuns double the employment of both labor and machines, i.e., \( L=50 \), \( K=80 \), we have the new output level

\[ Q = 25 \times 50 \times 80 = 100000 \], which is four times as large as the original output.

c. What is the total cost of the current production of PowerGuns in a month? What is the average cost to produce a shooting gun? Assuming the number of machines does not change, what is the marginal cost of producing one additional gun?

\[ TC = 3000 \times 25 + 6000 \times 40 = 315,000 \]

\[ AC = \frac{TC}{Q} = \frac{315000}{25000} = 12.6 \]
To compute the marginal cost, we solve this equation for the new level of labor employment (K is fixed), 25000 + 1 = 25 × L × 40, we get L = 25.001. So the marginal cost of producing one additional gun is, MC = 0.001 × $3000 = $3.00.

d. What is the law of diminishing returns? Does this production display this characteristic? Explain.

Law of diminishing returns states that the marginal product of a variable factor will eventually decline as the use of the factor is increased – also called the law of diminishing marginal product. No, it does not display the characteristics. For example, hold L fixed at 25. The marginal product of K is always 25L = 625 (i.e., for each unit increase in K, Q increases by 625 units).

5-21. Assume Kodak’s production function for digital cameras is given by

\[ Q = 100(L^{0.7}K^{0.3}) \],

where L and K are the number of workers and machines employed in a month, respectively, and Q is the monthly output. Moreover, assume the monthly wage per worker is $3,000 and the monthly rental rate per machine is $2,000. NOTE: Given the production function, the marginal product functions are

\[ MP_L = 70(L^{0.7}K^{0.3}) \] and \[ MP_K = 30(L^{0.7}K^{0.3}) \].

a. If Kodak needs to supply 60,000 units of cameras per month, how many workers and machines should it optimally employ?

Kodak should minimize the total cost of producing the 60,000 cameras, given the production function and input prices. Therefore, it should employ a labor-to-capital ratio such that the following condition holds:

\[ \frac{MP_K}{MP_L} = \frac{w_K}{w_L} \]

or

\[ \frac{(30L^{0.7}K^{0.3})}{(70L^{0.7}K^{0.3})} = \frac{2000}{3000} \]

When solved for the optimal L/K ratio, the equation yields: 9L = 14K,

or \( L = (14/9)K \). The optimal input ratio together with the production target produces a system of two equations in two unknowns (K and L), which has a unique solution. That is:
L = (14/9) × K

and

60,000 = 100(L^{0.7}K^{0.3})

Solving the system using standard techniques yields the optimal quantity of K and L; Kodak should employ $L^* = 685.04$ and $K^* = 440.38$.

b. What are the total cost and average cost of producing the quantity given in (a)?

The total cost of producing any quantity Q is obtained by using the following accounting equation:

$$TC(Q) = w_K K(Q) + w_L L(Q).$$

Substituting the data in the question for $w_K$ and $w_L$, and those in the previous answer for $L(Q)$ and $K(Q)$, one obtains the total cost of producing 60,000 digital cameras when the optimal K/L ratio is employed:

$$TC = 2000 \times 440.38 + 3000 \times 685.04 = $2,935,880$$

The average cost of production is simply equal to the total cost divided by the total output. Thus, the average cost for each camera is:

$$AC = \frac{2,935,880}{60,000} = $48.93$$

5-22. For simplicity, throughout this problem, assume labor (L), capital (K), and quantity produced (Q) can be infinitely divided – that is, it is fine to hire 3.3 workers, rent 4.7 machines, and/or produce 134.2 units. Answer the following questions, assuming the production function for DurableTires Corp. is $Q = L^{0.5}K^{0.5}$, where Q is the quantity of tires produced, L is the number of workers employed, and K is the number of machines rented.
a. What is the quantity of tires produced when the company employs 64 workers and 36 machines?

When \(K=36\) and \(L=64\), \(Q=24\):

\[
64^{1/3} \times 36^{1/2} = 24
\]

b. What are the average product of labor (L) and the average product of machines (K) when the input mix is the one given above? Clearly and concisely, please explain how you would interpret these numbers.

The average product of labor is \((24 / 64) = 0.375\); the average product of capital is \((24/36) = 0.667\).

Average product of a factor is equal to the total product divided by the number of units of the input considered. The average product of labor (capital) is the average output generated by one unit of labor (capital).

c. Continue to assume the input mix given above: What is the marginal product of labor (L), if the number of workers is increased by 1 unit? What is the marginal product of capital (K), if the number of machines is increased by 1 unit, instead? Clearly and concisely, please explain how you would interpret these numbers.

The marginal product of a input is the change in total output associated with a one unit change in that input, holding other inputs fixed.

\(L= 64+1; \ K =36; \ Q = 24.124; \ Marginal \ product \ of \ Labor = 24.124 – 24 =0.124\)

\(L= 64; \ K =36+1; \ Q = 24.331; \ Marginal \ product \ of \ Capital = 24.331 – 24 =0.331\)

An additional unit of labor (capital) increases production by 0.124 (0.331) units.

d. Does DurableTires’ production function display increasing, decreasing, or constant returns to scale? Explain. Would your answer change, if the production function were \(Q=L^{0.5}K^{0.5}\)? How? Explain.

If we increase all the inputs by one percent: \(L=64.64; \ K=36.36\). Output increases to 24.199, which represents a 0.83 percent increase from the original output of 24 units. Thus, output increases less then proportionally when all the inputs are increased by the same proportion, which implies that the production function displays decreasing returns to scale.
If production function were \( Q = L^{1/2}K^{1/2} \), then \( L= 64 \) and \( K = 36 \) results in \( Q = 48 \); and \( L= 64.64 \) and \( K = 36.36 \) results in \( Q = 48.48 \), which corresponds to a 1 percent increase in output. Therefore, the production function in this case \( (Q = L^{1/2}K^{1/2}) \) displays constant returns to scale, as output increases proportionally when all the inputs are increased by the same proportion.

e. Does DurableTires’ production function display increasing, decreasing, or constant returns to labor? Explain. Would your answer change, if the production function were \( Q=L^{1/3}K^{1/2} \)? How? Explain.

If we increase labor input by 1 percent, \( L= 64.64 \) and \( K = 36 \) results in \( Q = 24.08 \), which represents a 0.33 percent increase in output. Thus, the production function displays decreasing returns to scale to labor, since output increases less than proportionally as we increase the use of labor.

If the production function were \( Q = L^{1/3}K^{1/2} \), \( L= 64 \) and \( K = 36 \) results in \( Q = 48 \), while \( L= 64.64 \) and \( K = 36 \) results in \( Q = 48.24 \) which represents a 0.50 percent increase in output. Therefore, in this case also, the production function \( (Q = L^{1/3}K^{1/2}) \) displays decreasing returns to labor.

5-23. Answer the following questions, continuing to assume the production function for DurableTires Corp. is \( Q=L^{1/3}K^{1/2} \), where \( Q \) is the quantity of tires produced, \( L \) is the number of workers employed, and \( K \) is the number of machines rented. Moreover, assume the wage per unit of labor (\( w_L \)) is $50 and the rental price per machine is $200 (\( w_K \)).

a. What is the total cost of producing the quantity of tires you found in your answer to question (5-23)? And the average cost? Assuming the number of machines rented does not change, what is the marginal cost of producing one additional tire?

Given that \( w_L = $50 \) and \( w_K = $200 \), the Total Cost of producing 24 units of output is \((64\times50)+(36\times200) = $10,400\). The Average Cost is \((10,400/24=) $433.33\). For the calculation of Marginal Cost it is assumed that \( K \) cannot be immediately changed (\( K=36 \)). To increase production by one unit, \( Q = 24+1 \), given \( K = 36 \) and \( Q = L^{1/3}K^{1/2} \), \( L \) must increase to 72.34.

Then, the Total Cost becomes \((72.34\times50)+(36\times200) =) $10,817. Thus, the Marginal Cost of producing one additional unit is \((10,817 - 10,400 =) $417.\)
b. Given the production function above, the marginal product of labor and the marginal product of capital are \( \text{MP}_L = \frac{1}{3}(L^{2/3}K^{-1/2}) \) and \( \text{MP}_K = \frac{1}{2}(L^{1/3}K^{-1/2}) \), respectively. Given the wage and rental rate above, is DurableTires Corp. adopting an optimal input mix to produce the quantity of tires found in question (5-23a)? If yes, why? If not, why not, and how could DurableTires Corp. save money producing that same quantity of tires? Explain.

In order to achieve cost minimization, a company has to choose the input mix such that the slope of the isoquant is equal to the slope of the isocost line. That is:

\[-\frac{\text{MP}_L}{\text{MP}_K} = -\frac{P_L}{P_K}\]

where, given the production function and the input prices,

\[\frac{\text{MP}_L}{\text{MP}_K} = \frac{0.125}{0.333} = 0.375\]

\[\frac{P_L}{P_K} = \frac{50}{200} = 0.25\]

In order to increase profitability for an output of 24 units, one needs to determine the optimal quantity of \( K \) and \( L \) that, given \( Q = L^{1/3} \times K^{1/2} \), will yield \( Q = 24 \). Then, using the optimal \( (L,K) \) combination, and given that \( w_L = \$50 \), and \( w_K = \$200 \), one can calculate the minimum cost of production for 24 units of output.

When we plot \( L \) and \( K \) quantities against cost of production (see graph below) we see that cost of production decrease along the arrows indicated in the graph. The minimum total cost is achieved when \( L=81.6 \) and \( K=30.6 \). You can get this by solving \( L \) and \( K \) from the two equations (1) \( \frac{\text{MP}_L}{\text{MP}_K} = 0.25 = \frac{P_L}{P_K} \) and (2) \( 24 = L^{1/3} \times K^{1/2} \).

Intuitively, the ratio of the \( \text{MP}_K \) and \( \text{MP}_L \) was too big, increasing the denominator and decrease the numerator would make the ratio approach 0.25. Doing so implies employing more \( L \) and less \( K \).

The total cost of producing 24 units with this input mix is \$10,203, which is less than the \$10,400 calculated earlier, when the company employs \( L=64 \) and \( K=36 \). Notice that using \( (L,K) = (81.6, 30.6) \):
\[ MP_K = 0.390 \]
\[ MP_L = 0.098 \]
\[ \frac{MP_L}{MP_K} = 0.25 = \frac{P_L}{P_K} \]

c. What happens to the optimal input mix you found in question (5-23) if the government introduces a tax that raises the cost of labor to $150 per worker? Explain.

After a tax on labor is introduced, workers become relatively more expensive. Thus, for the same output level, the company will shift toward an input mix that is relatively more capital intensive than the original one.
5-24. Assume DurableTires Corp. faces the following demand curve, \( P = 250 - 0.1Q \). If DurableTires’ marginal cost is constant at $35, how many tires should it produce in order to maximize its profits? What’s DurableTires’ profit in this case? Should the elasticity of demand be greater, equal, or less than 1 at the profit-maximizing price and quantity? Explain (hint: you may use a graph to support your argument).

The demand is \( P = 250 - 0.1Q \), and the marginal cost is \( MC = $35 \). Thus:

\[
TR = Q \times (250 - 0.1 \times Q)
\]

And

\[
MR = 250 - 2 \times 0.1 \times Q.
\]

For profit maximization, a firm needs to ensure that \( MR = MC \). That is:

\[
250 - 2 \times 0.1 \times Q = 35
\]

\[
Q = 1,075
\]

The firm should charge \( P = $142.5 \) for this quantity. And the profit will be:
Profit = \((142.5 - 35) \times 1,075\) = $115,562.5

Because this is the profit maximizing quantity for a firm having a positive marginal cost, the elasticity of demand should be greater than 1. See next graph.