CHAPTER 4

DEMAND

CHAPTER SUMMARY

This chapter builds on a brief introduction in the previous chapter by providing a more detailed analysis of product demand. The chapter covers demand functions and demand curves. It introduces elasticities and the properties of linear demand curves. The relation between total revenue and price is examined. The chapter also provides a brief introduction to network effects, the product-attribute model, product life cycles, and demand estimation. An appendix covers point elasticities, marginal revenue for linear demand curves, and log-linear demand curves.

CHAPTER OUTLINE

DEMAND FUNCTIONS
DEMAND CURVES
Law of Demand
Managerial Application: Learning the Law of Demand the Hard Way
Elasticity of Demand
Managerial Application: Increased Foreign Competition and Demand Elasticities
Calculating Price Elasticities
Price Changes and Total Revenue
Determinants of Price Elasticities
Academic Application: Price Elasticities
Managerial Application: Demand Elasticities and Airline Pricing
Linear Demand Curves
Total Revenue
Marginal Revenue
Profit Maximization
OTHER FACTORS THAT INFLUENCE DEMAND
Prices of Related Products
Complements versus Substitutes
Managerial Application: Complementarity between Computer Hardware and Software
Managerial Application: Derived Demand
Cross Elasticities
Academic Application: Estimates of Cross Elasticities
Income
Normal versus Inferior Goods
Managerial Application: Russian Cola Wars
Chapter 04 - Demand

Income Elasticities
   Academic Application: Estimates of Income Elasticities
   Managerial Application: A Pampered Dog Loses His Stylist

Other Variables

INDUSTRY VERSUS FIRM DEMAND
   Industry Demand Curves
     Defining Industry and Market Area
     Managerial Application: 9/11 Causes Massive Shifts in Demand Curves
     Managerial Application: Store Layout Affects Demand
     Managerial Application: Demand Elasticity for Gasoline

NETWORK EFFECTS
   Managerial Application: eBay and Network Effects

PRODUCT ATTRIBUTES
   Managerial Application: Understanding What Consumers Want

PRODUCT LIFE CYCLES
   Managerial Application: First-Mover Advantages and Financial Innovation

DEMAND ESTIMATION
   Interviews
     Managerial Application: Using Technology to Assess Demand
   Price Experimentation
   Statistical Analysis
     Academic Application: On Estimating Demand Curves for Common Stocks
     Omission of Important Variables
     Multicollinearity
     Identification Problem

Implications

SUMMARY

APPENDIX: DEMAND
   Point Elasticities
   Marginal Revenue for Linear Demand Curves
   Marginal Revenue and Demand Elasticity
   Log-Linear Demand Functions
TEACHING THE CHAPTER

This chapter reiterates the fundamentals of demand. In upper level courses that require students to complete a managerial economics or intermediate microeconomics course as a prerequisite, extensive coverage might not be as necessary, however it is essential for students to know the basic concepts in the chapter since they relate to material covered in later chapters. For example, the properties of linear demand curves are relevant in Chapters 17 (transfer pricing) and 19 (vertical integration and outsourcing). There are numerous Managerial Applications within the chapter that are used to illustrate key concepts (such as the relationship between elasticity and changes in revenue due to price changes) that can be used to generate class discussion of the topics rather than relying on lecturing. If students have only a limited economics background, the quantitative analysis will be new, however the tools are not too advanced so most students should not struggle with learning them.

The Self-Evaluation Problems review the quantitative concepts presented in the chapter and provide a good way to determine whether students fully understand the concepts. There are numerous Review Questions at the end of the chapter that cover both the concepts and the quantitative tools of the chapter. Students might find these questions difficult if they have not worked through the Self-Evaluation Problems first. Instructors will likely want to review some of these questions to ensure students understand the material before assigning the Analyzing Managerial Decisions scenarios.

There are three Analyzing Managerial Decisions scenarios presented in this chapter. The first, “Setting Tuition and Financial Aid at Ursinus College”, asks student to evaluate whether demand for a college education adheres to the law of demand. Students are presented an apparent violation of the law of demand and asked to determine if this explanation seems plausible. The second, “Demand Curve for an Indian Electronics Product,” is a quantitative analysis that asks students to derive the equation for the firm’s demand curve and to determine whether demand is elastic or inelastic at the prices referenced. Using this information, students are then asked to analyze whether the current price maximizes the bonus that can be earned under the compensation plan and whether the individual goals are aligned with the firm’s profit maximization goal. This question is a good precursor of the material that will be presented later in the text. The third scenario, “Personal Video Recorders” is a more comprehensive scenario that asks students to analyze how advertising might be affected by the use of digital video recorders. This scenario includes a quantitative analysis. (See the Solutions Manual for the answers to these problems).
REVIEW QUESTIONS

4–1. What is the difference between a demand function and a demand curve?

A demand function is a mathematical representation of the relation between the quantity demanded of a product and all factors that influence this demand. A demand curve pictures how many units will be purchased at each possible price, holding all other factors fixed.

4–2. How will each of the following affect the position of the demand curve for videocassette recorders (VCRs)?

a. An increase in the price of VCR tapes.

An increase in the price of VCR tapes will shift the demand curve for video recorders to the left (demand will decrease since the price of a complement has increased).

b. A decrease in the price of VCRs.

A decrease in the price of VCR’s does not shift the position of the demand curve. Rather it results in an increase in the quantity demanded (movement along the curve).

c. An increase in per capita income.

An increase in personal income is likely to shift the demand curve for VCRs to the right (VCRs are probably normal goods).

d. A decrease in the price of movie tickets.

The decline in movie ticket prices might shift the demand for VCRs to the left (demand will decrease if movies and VCRs are substitutes).

4–3. If the demand for a product is inelastic, what will happen to total revenue if price is increased? Explain.

The total revenue will increase. When demand is inelastic, a given percentage increase in price corresponds to a smaller percentage decrease in quantity demanded. Thus total revenue, which is equal to price times quantity, must increase.
4–4. What sign are the cross elasticities for substitute products? Explain.

The cross elasticities between substitutes are positive. Cross elasticities measure the percentage change in the quantity demanded for a good given a percentage change in the price of another good. When the price of a good increases, the demand for substitute products increase.

4–5. Distinguish between normal and inferior goods.

The demand for normal goods increases with income, while the demand for inferior goods decreases with income.

4–6. Is it true that a normal good must have an income elasticity that is more than one? Explain.

No – being a normal good means that the quantity demanded of the good increases with income. Therefore, the income elasticity simply has to be positive (> 0).

4–7. Suppose that the price of Product A falls from $20 to $15. In response, the quantity demanded of A increases from 100 to 120 units. The quantity demanded for Product B increases from 200 to 300. Calculate the arc cross elasticity between Product B and Product A. Is B a substitute or complement for A? Explain. Does Product A follow the “law of demand?” Explain.

Cross Elasticity between B and A = \( \frac{(300-200)}{(15-20)} \times \frac{\left(\frac{20+15}{2}\right)\left(\frac{300+200}{2}\right)}{1} = -2.33 \). The two products are complements since they have a negative cross elasticity. An increase (decrease) the price of product A causes a decrease (increase) in the demand for B.

The law of demand says that the quantity demanded for a good increases as its own price falls. This is true in the example for product A.

4–8. How can cross elasticities be used to help define the relevant firms in an industry?

Firms with high positive cross elasticities are strong substitutes and thus might be considered to be competing in the same industry.

4–9. Suppose the price of heating oil increases significantly. Discuss the likely short-run and long-run effects.
An increase in the price of heating oil is likely to decrease the quantity demanded for heating oil (people buy less at higher prices). The long-run effects are likely to be larger than the short-run effects for a permanent increase in the price. Over the long-run, people will invest in more efficient heating equipment or equipment that uses natural gas or electricity; they will insulate their homes better, and so on. These actions will further decrease the demand for heating oil.

4–10. The Alexander Machine Tool Company faces a linear demand curve. Currently, it is selling at a price and quantity where its demand elasticity is 1.5. Consultants have suggested that the company expand output because it is facing an elastic demand curve. Do you agree with this recommendation?

The elastic demand implies that total revenue will increase with an expansion in output (which must be sold at a lower price). However, this does not mean that Alexander Machine Tool should necessarily expand output. Alexander wants to maximize profits. Whether or not expansion is sensible depends on not only what happens to revenue, but also what happens to cost. There is not enough information in the problem to tell whether Alexander should increase output. Profit maximization is discussed in more detail in chapters 5 and 6.

4–11. For three years in a row, income among consumers has increased. Alexander Machine Tool has had sales increases in each of these three years. Does Alexander Machine Tool produce inferior or normal goods? Forecasts predict that income will continue to rise in the future. Should Alexander Machine Tool anticipate that demand for its products will continue to rise? Explain.

The increase in sales might have been motivated by factors other than the increase in income (other factors have not been held constant). Therefore, it is not clear whether Alexander is producing an inferior or normal good. Without a more careful analysis, Alexander should be careful in forecasting that sales will increase with the expected increase in income.

4–12. The cross elasticity between product A and product B is 10. Do you think that product A is likely to face an elastic or inelastic demand curve? Explain.

The cross elasticity of 10 suggests that B is a strong substitute for A. Thus, A’s demand curve is likely to be relatively elastic. If A increases its price, it is likely to lose sales to B.
4–13. Vijay Bhattacharya is interested in estimating the industry demand curve for a particular product. He has gathered data on historical prices and quantities sold in the industry. He knows that the industry supply curve has been stable over the entire period. He is considering estimating a regression between price and quantity and using the result as an estimate of the demand curve. Do you think this technique will result in a good estimate of the demand curve? Explain.

The regression will not produce a good estimate of the demand curve due to the identification problem. The data on price and quantity are equilibrium combinations observed in the marketplace; they reflect the positions of both the supply and demand curves. Given that the supply curve has been stable, changes in price and quantity over the period reflect shifts in the demand curve. In this case, the regression is likely to generate a better estimate of the supply curve than the current demand curve.

4–14. Maria Tejada, a civil engineer uses data on population trends to forecast the use of a particular highway. Her forecasts indicate severe road congestion by the year 2010. She suggests building a new road. Comment on this approach.

This demand forecast is likely to be based on current costs people face in using the highway (tolls, waiting times, and so forth). As highway use increases, the increased congestion will motivate some people not to use the road during busy time periods. For instance, they might drive to work early or take the train. These types of adjustments are likely to lower congestion during peak hours below what is forecast. Also the government might encourage more efficient use of the highway by appropriate tolls, and so forth. Tolls will help to internalize some of the increased waiting time that a commuter imposes on other commuters and thus will decrease the demand for road use. These types of actions might be preferred to building more highways.

4–15. Alexander Machine Tool faces the demand curve: \[ P = 70 - 0.001Q \]. What price and quantity maximize total revenue? What is the price elasticity at this point?

Total revenue is maximized at the midpoint of a linear demand curve. In this example, the quantity and price at the midpoint would be 35,000 and $35. The price elasticity at this point is one (unitary elasticity).

4–16. Studies indicate that the income elasticity of demand for servants in the United States exceeds 1. Nevertheless, the number of servants has been decreasing during the last 75 years, while incomes have risen significantly. How can these facts be reconciled?
Factors other than income were not constant over the past 75 years. For example, increases in the prices of servants and decreases in the price of substitutes (for example, washing machines, microwave ovens, and vacuum cleaners) would explain why demand for servants has declined even though incomes have risen.

4–17. Prior to a price increase, the price and quantity demanded for a product were $10 and 100, respectively. After the price increase, they were $12 and 90.

a. Calculate the arc elasticity of demand.

Using equation 4.5 in the book, the arc elasticity is .579.

b. Is the demand elastic or inelastic over this region?

Demand is inelastic (elasticity is less than 1).

c. What happened to total revenue?

Total revenue increased ($1,000 to $1,080).

4–18. Define marginal revenue. Explain why marginal revenue is less than price when demand curves slope downward.

Marginal revenue is the change in total revenue given a unitary change in price. Marginal revenue is less than price, when demand curves slope downward, because the price has to be lowered to increase sales. When quantity expands, all units must be sold at a lower price.

4–19. In 1991, Rochester, New York, had a serious ice storm. Electric power was out in houses for days. The demand for power generators increased dramatically. Yet the local merchants did not increase their prices, even though they could have sold the units for substantially higher prices. Why do you think the merchants adopted this policy?

The analysis in chapter 4 suggests that the merchants would have increased the price. Given the inelastic demand, the merchants could have increased their profits by increasing the price of the generators. The analysis in the chapter, however, focuses on a single period. The merchants might not have wanted to take advantage of the situation because it would have made customers angry and less likely to patronize them in the future.
4–20. Seven teenagers, four boys and three girls, were given $200 each to go on a shopping spree. An advertising agency, which specializes in youth markets, gave the teens the money. An account executive accompanied the teens while they were shopping. Not only did the agency want to learn what they bought, but also what they talked about to see what was on their minds. “It’s not so much to stay in tune with trends, because trends are elusive. It’s more what’s really happening with teens and what’s important to them.”

a. Discuss the trade-offs between sample size (7 teens), cost, and reliability of what is learned from this experiment.

Increasing the sample size would cost more. First, each additional teenager would be given $200. More importantly, the agency might have to add account executives to work with the teens. If not, the amount of detailed information that could be collected would decrease. Increasing the sample size provides a broader set of teenagers and is more likely to provide more general and accurate information about teen preferences. Collecting detailed information, however, is expensive. Thus, there is a tradeoff between cost and reliability.

b. An agent accompanied the teens while they were shopping. Why didn’t the ad agency avoid this expense and just look at what the teens bought?

The agency wanted to understand the thought process that the teenagers went through in selecting their purchases. For example, what alternatives did they consider? What were the most important factors that led to the purchase decision, and so on? Knowing this type of information is potentially important in making advertising, new product, and other marketing decisions.

4–21. Southwest Airlines estimates the short-run price elasticity of business fares to be 2 and the long-run elasticity to be 5. Is ticket demand more elastic in the short-run or long-run? Does this seem reasonable? Explain.

Fares are more elastic in the long run.

Suppose Southwest raises prices. In the short-run some travelers will switch to other airlines. However, corporate contracts and frequent flyer miles will cause some people to stay with Southwest.

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1 “Teens Track Retail Trends for Ad Agency,” Democrat and Chronicle (September 5, 1999), 1E.
In the longer run, businesses may switch to alternative forms of transportation, conduct more meetings by phone, or arrange more trips farther in advance to get lower fares. They may also acquire corporate jets.

4-22. Gasoline prices increased substantially in 2004 and 2005. What adjustments did people make to minimize the long-term effects of this price increase?

People made a variety of adjustments. For example, some purchased new, more fuel-efficient cars. Some invested in solar driven or hybrid vehicles. Others moved closer to work or other primary destinations.

4-23. Assume that demand for product A can be expressed as \( Q_A = 500 - 5P_A + 3P_B \) and demand for product B can be expressed as \( Q_B = 300 - 2P_B + P_A \). Currently, market prices and quantities for these goods are \( P_A = 5 \), \( P_B = 2 \), \( Q_A = 481 \), and \( Q_B = 301 \).

a. Suppose the price of product B increases to 3. What happens to the quantity demanded of both products?

The quantity demanded for product A increases from 481 to 484, while the quantity demanded for product B falls from 301 to 299.

b. Calculate the arc cross-elasticity between product A and product B using prices for product B of 2 and 3.

The arc cross-elasticity calculated between the two points implied by prices for good B of 2 and 3 is:

\[
\frac{(484 - 481)/(481 + 484)/2}{[(3 - 2)/(2 + 3)/2]} = \frac{0.00621}{0.04} = 0.16
\]

c. Are these goods substitutes or complements?

The two goods are substitutes: when the price of one good rises by a percent, the quantity demanded of the other good increases (in this case by less than one percent).

4-24. The Zenvox Television Company faces a demand function for its products which can be expressed as \( Q = 4000 - P + 0.5I \), where \( Q \) is the number of televisions, \( P \) is the price per television, and \( I \) is average monthly income. Average monthly income is currently equal to $2,000. Answer the following questions.
a. Graph the demand curve (sometimes called the “inverse” demand curve) faced by Zenvox at the current income level. Be sure to label this and all graphs you draw carefully. On the same graph, depict marginal revenue. At what price and quantity is Zenvox’s total revenue maximized? What is the marginal revenue at this point? Show the calculation.

Reversing the demand function to get \( P \) on the left-hand side, we get \( P = 4,000 - Q + .5I \). Plugging the stated value for income ($2,000) into the right-hand side, we get the inverse demand curve for this income level as \( P = 5,000 - Q \). This is plotted with quantity on the x-axis and price on the y-axis, as a straight line with a y-intercept of 5,000 and a slope of -1. The x-intercept will equal 5,000.

Total revenue (for this value of \( I \)) will be \( TR = PQ = (5,000 - Q)Q = 5,000Q - Q^2 \). Marginal revenue is given by \( MR = 5,000 - 2Q \). This will be graphed as a straight line with y-intercept of 5,000 and a slope of -2. The x-intercept will be 2,500. The following graph depicts both curves:

Since the demand curve is linear, the point at which total revenue is maximized is at the midpoint, which corresponds to a price of $2,500 and quantity of 2,500. Total revenue at this point is $6,250,000. Marginal revenue is equal to zero at this point:

\[
MR = 5,000 - 2Q = 5000 - 2(2500) = 0.
\]

b. What is the price elasticity of Zenvox’s demand function at the price and quantity derived in part (a)? Explain what this value means in words.
The price elasticity at the midpoint of a linear demand curve is 1. In words, this value means that a one percent increase (decrease) in price is met with a one percent decrease (increase) in quantity demanded.

c. Why might Zenvox choose to produce at a price and quantity different than that derived in part (a)?

Companies seek to maximize profits, which is generally different than maximizing total revenue. The company will take into account the costs it faces in determining its optimal price and quantity.

4-25. According to an article in Forbes (March 2001) teen cigarette smoking declined significantly between 1975 and 2000. The most dramatic decline occurred in the years 1975–1981. Since then teen smoking has increased in some years and declined in others. Between 1975 and 1981 there was a slight decrease in the price of cigarettes. Thus the dramatic decline in smoking is not attributable to an increase in cigarette prices. One theory is that the significant increase in gasoline prices over this period motivated many teens not to smoke.

a. Discuss how a rise in gasoline prices might affect the demand for cigarettes among teens.

One way to answer the question is to focus on the standard effects in our analysis of consumer behavior in a two-good world: A rise in the price of gasoline holding the price of cigarettes constant will have two effects: teens will tend to substitute away from consuming gasoline to consuming cigarettes as cigarettes become relatively cheaper than they used to be (substitution effect), and teens will face a lower “real” income due to the increase in gasoline prices (income effect). The total effect on cigarette consumption will be uncertain, but if gasoline is a large enough expenditure for teens the income effect may dominate and cause lower consumption of cigarettes.

Another acceptable method of answering the question is as follows:

Cigarettes and gasoline might be complementary products (e.g., people tend to smoke when they drive). In this case, a rise in gasoline prices would result in a decline in the demand for cigarettes (teenagers drive less and smoke less). Also teenagers may spend a higher proportion of their budget on gasoline with higher gasoline prices. Thus there is less money left to spend on other goods such as cigarettes.
b. Suppose there are two goods in the world, cigarettes and gasoline. Draw a figure that shows how an increase in gasoline prices can result in a decline in both gasoline and cigarette consumption. Use the standard consumer behavior graph with budget lines and indifference curves. Be sure to label your figure appropriately.

![Graph showing the relationship between gasoline and cigarette consumption](image)

In the above figure, suppose the initial budget line, before the gas price increase, is Budget Line 1, and the gas price increase causes the line to shift to the position of Budget Line 2. The optimal consumption bundle shifts from A to B, which has lower consumption of both goods.

c. In the late 1990s the price of cigarettes increased from $2.50 per pack to $3.25 per pack. In one community during this time period, the number of packs of cigarettes consumed by teenagers fell from 10,000 to 9,000. Assume that everything except cigarette prices remained the same. Calculate the arc price elasticity among teens between these price points.

The arc price elasticity is $\text{abs}\left(\frac{-1000}{.75}\right)\times\frac{2.875}{9500}=.4035$.

d. Calculate the total expenditures on cigarettes by teens in part (c) both before and after the price increase. Did total revenue increase or fall? Discuss how this answer is implied by the arc elasticity that you calculated in part (c).

Before price change: $2.50\times10,000=25,000$

After price change: $3.25\times9,000=29,250$

Total revenue increased. This is implied by the arc elasticity because when elasticity is less than one, an increase in price increases total revenue. When elasticity is less than one, a 1 percent increase in price leads to a less than 1 percent decrease in total quantity, so total revenue, which is price times quantity, increases.
4-26. In an article appearing in the Dow Jones News Service on February 5\textsuperscript{th}, 2004, the agency cites Saudi Arabia’s concern about the production of oil by the OPEC cartel. Assume the current daily demand for OPEC’s oil is given by the following equation:

\[ P = 50 - 0.001Q \]

where \( P \) is the price per barrel (ppb) and \( Q \) is the quantity of barrels sold daily (in thousands). Moreover, suppose the marginal cost of producing a barrel is constant at zero.

a. Would it surprise you to learn that OPEC’s declared objective is to sell 25 million barrels a day for an average price of $25 per barrel? Why or why not? Explain. You may use a graph to support your argument.

It is not surprising that the OPEC’s declared objective is to sell 25MM barrels a day for an average price of $25 per barrel. As the picture above shows, this quantity maximizes the OPEC’s total revenue from the production of oil, given the demand curve in the question. More importantly, given that \( MC=0 \), it also maximizes OPEC’s profit from selling oil.
In particular, the total revenue function for the OPEC’s oil is:

\[ TR = P \times Q \]

or

\[ TR = (50 - 0.001 \times Q) \times Q \]

or \[ TR = 50Q - 0.001Q^2 \]

The corresponding Marginal Revenue curve is (notice: \( MR \) has the same intercept and twice the slope of the original inverse demand curve, as pointed out in class):

\[ MR = 50 - 2 \times 0.001 \times Q = 50 - 0.002Q \]

As explained in class, profit is at the maximum when \( MR = MC \). Thus, by explicitly solving the previous condition one obtains the \( Q \) that maximizes profit. Given that \( MC = 0 \) by assumption, the condition reduces to the following:

\[ MR = 0 \]

or

\[ 50 - 0.002Q = 0 \]

The solution to the previous equation is \( Q = 25,000 \). For this quantity, OPEC can charge (at most) \( P = $25 \).

b. Assume that after OPEC’s meeting this week, the new demand for OPEC oil will be given by: \( P = 40 - 0.001Q \). Would OPEC’s stated objective (25 million barrels at an overall price of $25) be attainable after this change? Explain. Assume OPEC ignores the demand shift. What’s the maximum price per barrel they can charge if they decide to keep producing 25 million barrels per day? What is the profit in this case?

As the figure below shows the OPEC’s declared objective is not feasible after the demand change. If OPEC wants to charge $25 per barrel, it will have to cut back production to 15MM barrels per day. If OPEC wants to keep selling 25MM barrels per day after the demand change, it will have to lower the price per barrel to $15 per barrel. In this latter case, the OPEC’s profit is going to be $375,000,000 per day.
c. Now suppose that OPEC recognizes that demand has changed (as in (b)) and wants to maximize profits. What is the daily quantity they should supply? At what price? What is the profit in this case? What is the price elasticity of demand at this price/quantity combination? Explain.

The objective of selling 25MM barrels per day is not optimal after the demand shift. Indeed, after the change takes place, the optimal daily quantity is 20MM barrels, which should be sold at a price of $20 per barrel – as shown in the picture. The optimal price/quantity combination is obtained following the same steps shown in the answer to part (a).

The daily profit corresponding to this price/quantity combination is $400,000,000, which is higher than the profit OPEC would earn if it kept production at 25MM barrels per day ($375,000,000 from part (b) of the question).

The price elasticity of demand is equal to 1. This is because the MC of production is constant at zero and, thus, profit maximization is equivalent to revenue maximization. As described in class, if a firm is to maximize total revenue, it should choose the price/quantity combination such that the price elasticity of demand is equal to 1.
4-27. As a result of strikes in Canada the world price of nickel rose by 20 percent in December. Over the same period, the quantity demanded of nickel decreased from 10,000,000 to 8,500,000 pounds worldwide. The world price of nickel was 70 cents per pound before the strikes.

a. Show graphically the effect of Canadian strikes on the market for nickel.

b. Given the information above, what’s the price elasticity of the world demand for nickel over the relevant price range?

The formula for the arc-elasticity is the following:

$$\eta_P(Q) = \left| \frac{\Delta Q/(Q_1+Q_2)/2}{\Delta P/(P_1+P_2)/2} \right|$$

Substituting the figures given in the question in the previous formula gives:

$$\eta_P(Q) = \left| \frac{-1,500,000 / 9,250,000}{0.14 / 0.77} \right| = 0.891892$$

c. Did the total expenditure for nickel increase, decrease, or remain constant after the strikes? How is this consistent with your answers to parts (a) and (b)? Explain clearly and concisely.

The total expenditure for nickel increased from $7,000,000 (0.7*10,000,000) before to $7,140,000 (0.84*8,500,000) after the strikes. This is consistent with the elasticity of demand being less than one over the relevant price range (i.e. demand being inelastic). As discussed in class, in the inelastic portion of the demand ($\eta<1$), total revenue/expenditure increases as the price increases.

4-28. Assume the demand curve for gasoline is given by the following equation:
P = 10 – 0.005Q, where P is the price per gallon and Q is the quantity of gasoline in gallons. Assume that the only supplier of gasoline in the region is General Gasoline Co. and that the marginal cost of production is constant at zero.

a. If the company is currently charging $4 a gallon, is it maximizing profit? If so, prove it. If not, find out the price that maximizes its profit, and compare the profits at the two prices.

MR = 100 – 2× 0.005Q MR = MC = 0, solve for Q∗ = 10,000. Substitute this into the demand curve and get the price, P∗ = 100 – 0.005(10,000) = $50. This is less than the $60 the company is charging now. Profit with $60 can be calculated as the following. First get the quantity in equilibrium. $60 = 100 – 0.005Q. Q = 8000. Profit is then $60 (8000) = $480,000. Profit with $50 is $50(10,000) = $500,000 > $480,000. Setting prices to be $50 is indeed more profitable than setting the price to be $50.

b. Discuss the likely effect of the introduction of a fuel-efficient car in the region, i.e. what would happen to the equilibrium quantity. Show the changes on a graph that displays (you don’t need to show actual numbers) General Gasoline’s pricing solution and explain.

Fuel efficient cars will decrease the demand for gasoline. The dashed lines are the demand curve and MR curve after the introduction of the fuel efficient car (which reflects a decrease in the demand for gasoline). The profit-maximizing quantity would be the intersection of the MR curve with the x-axis, which is the MC curve. The quantity in equilibrium is lower than that before.

4-29. The accompanying chart presents data on the price of fuel oil, the quantity demanded of fuel oil, and the quantity demanded for insulation.
Fuel Oil

<table>
<thead>
<tr>
<th>Price per gallon</th>
<th>Quantity demanded (Millions of gallons)</th>
<th>Insulation</th>
<th>Quantity demanded (Millions of tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.00</td>
<td>100</td>
<td>$5.00</td>
<td>30</td>
</tr>
<tr>
<td>$5.00</td>
<td>90</td>
<td>$7.00</td>
<td>35</td>
</tr>
<tr>
<td>$7.00</td>
<td>60</td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>

a. Calculate the price elasticity (arc-elasticity) of demand for fuel oil as its price rises from 30 cents to 50 cents; from 50 cents to 70 cents. Calculate the change in total revenue in the two cases. Explain how the changes in revenue relate to your estimated elasticities.

\[ \eta = 0.2105 \text{ for price change from 30 cents to 50 cents} \]
\[ \eta = 1.2 \text{ for price change from 50 cents to 70 cents} \]

Total revenue increases (decreases) with a price increase when demand is inelastic (elastic). This is what happens here. Revenue increases in the first case ($3,000 to $4,500) and falls in the second case ($4,500 to $4,200).

b. Calculate the arc cross-elasticity of demand for insulation as the price of fuel oil rises from 50 cents to 70 cents. Are fuel oil and insulation substitutes or complements? Explain.

\[ \eta_{P,I}Q_{fuel} = 0.2 > 0, \text{ fuel oil and insulation are substitutes since the cross-elasticity is greater than zero. Higher fuel oil prices lead to an increase in demand (and consumption) for insulation.} \]

4-30. Japan has 4,350 miles of expressway – all toll roads. In fact, the tolls are so high that many drivers avoid using expressways. A typical 3 hour expressway trip can cost $47. A new $12 billion bridge over Tokyo Bay that takes 10 minutes and costs $25 rarely is busy. One driver prefers snaking along Tokyo’s city streets for hours to save $32 in tolls.\(^2\) Assume that the daily demand curve for a particular stretch of expressway is:

\[ P = 800 \text{ yen} - 0.16 Q. \]

a. At what price-quantity point does this demand curve have a price elasticity of one?

With a linear demand curve, unitary price elasticity occurs at the midpoint of the demand curve. Recall, this is the point where marginal revenue equals zero. To derive the MR curve, take twice the negative slope,

\[ MR = 800 \text{ yen} - 0.32Q = 0 \]

\[ 0.32Q = 800 \]

\[ Q = 2500 \]

\[ P = 800 - 0.16(2500) \]

\[ P = 400 \text{ yen} \]

Alternatively, you can derive the midpoint of the demand curve by taking half the intercept (800 yen), or \( P = 800/2 = 400 \) and solve for \( Q \) when \( P = 400 = 800 - 0.16Q \). In this case, we get \( 0.16Q = 400 \), or \( Q = 2500 \).

b. Assume the government wishes to maximize its revenues from the expressway, what price should it set? And how much revenue does it generate at this price?

Maximum revenue occurs at unitary price elasticity, \( P = 400 \text{ yen} \) and \( Q = 2500 \text{ cars} \).

Total Revenue = \( P \times Q = 1,000,000 \text{ yen per day} \)

c. Suppose that traffic engineers have determined that the efficient utilization of this particular toll road is 4,000 cars per day. This traffic level represents an optimum tradeoff between congestion (with its associated reduction in speeds and increase in accidents) between expressways and surface roads. If 4,000 cars per day is the socially efficient utilization of the toll road, what price should be set on the toll road? And how much revenue is collected by the government?

If 4,000 cars per day is the socially efficient utilization of the expressway, then the price should be:

\[ P = 800 \text{ yen} - 0.16(4000) \]

\[ P = 160 \text{ yen} \]

Revenue = \( P \times Q = 640,000 \text{ yen} \)

d. Which price, the one in part b, or the one in part (a) would you expect the government to set?
If the expressway users are organized and have their own special interest lobby, they might put pressure on the government to lower the tolls. On the other hand, government officials looking to reduce budget constraints would like to keep the tolls high, probably closer to the price in part (b), than in part (c).